MATH 2460 EXAM 4

NAME Key — GRADE -OUT OF **15 PTS**

Answer the following questions correctly (*no decimal answer, except in* 1(b)) for a full credit.

1. (**2pts**) Given the term of the series $\frac{ln2}{2} + \frac{ln3}{3} + \frac{ln4}{4} + \ldots +$, answer the following:

(a) Write the n^{th} term or the general expression of the series.

$$a_n = \frac{\ln(n)}{n} \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{\ln n}{n}$$

(b) Find the first *three* terms of the sequence of partial sums. (Round your answers to *four decimal places.*)

$$S_1 = \frac{\ln(2)}{2} \sim .3466$$

$$S_2 = \frac{\ln 2}{2} + \frac{\ln 3}{3} \sim .7128$$

$$S_3 = S_2 + \frac{\ln 4}{4} \sim 1.0594$$

2. (**2pts**) Find the *sum* of the convergent series: $\sum_{n=0}^{\infty} 7\left(\frac{3}{4}\right)^n$

$$S = \frac{7}{1-\frac{3}{4}} = \frac{7}{\frac{1}{4}} = \boxed{28} \quad \text{since } \frac{3}{4} < 1$$

3. (**3pts**) Which, if any, of the following series are telescoping, geometric, $p-$series, harmonic, alternating or other.

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (c) $\sum_{n=0}^{\infty} \frac{\pi}{2^n}$ (d) $\sum_{n=0}^{\infty} (-1)^n \frac{5^n}{n!}$

(e) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ (f) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (g) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Write down the corresponding letter (or *none*) for the series here:

Telescoping f Geometric c $p-$series b, g

Harmonic a Alternating d Others e

4. (**2pts**) A ball is dropped from a height of **9** feet. Each time it drops, it rebounds and its new height is **3/4** of its previous height. Find the *total vertical distance* traveled by the ball until it stops.

$$\text{Distance} = 9 + 2\cdot 9\left(\frac{3}{4}\right) + 18\left(\frac{3}{4}\right)^2 + 18\left(\frac{3}{4}\right)^3 + \cdots$$

$$= 9 + \sum_{n=1}^{\infty} 18\left(\frac{3}{4}\right)^n \quad \text{or} \quad \sum_{n=0}^{\infty} 18\left(\frac{3}{4}\right)^n - 9$$

$$= \frac{18}{1-\frac{3}{4}} - 9 = \frac{18(4)}{1} - 9$$

$$= 72 - 9$$

$$\boxed{\text{Distance} = 63\text{ ft}}$$

5. (**2pts**) Does the integral $\int_0^\infty e^{-x}\,dx$ converge? *Justify* your answer after evaluating it. (<u>show</u> work for a full credit!)

$$\lim_{b\to\infty}\left[-e^{-x}\right]_0^b = \lim_{b\to\infty}\left[-e^{-b}\right] + 1$$

$$= 1 \quad (\text{exists})$$

So $\int_0^\infty e^{-x}\,dx$ converges

6. (**4pts**) Which, if any, of the following series diverge, converge, converge absolutely, converge conditionally. Justify your answer by showing your work (*after stating the test used!*).

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$ $= \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{\frac{1}{2}}}$ Let $a_n = \frac{1}{\sqrt{n}}$ $a_{n+1} = \frac{1}{\sqrt{n+1}}$

AST Alternating Series Test

- $a_{n+1} \leq a_n \ \forall n \geq 1$
- $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right) = 0 \to \sum (-1)^n \frac{1}{\sqrt{n}}$ converges conditionally, since $\sum \frac{1}{n^{\frac{1}{2}}}$ diverges By p-series test $(P = \frac{1}{2} < 1)$

(b) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2}$ $= \sum \frac{(-1)^n}{n^2}$

$\sum \frac{1}{n^2}$ converges by p-series test with $p = 2 > 1$, so $\sum \frac{\cos(n\pi)}{n^2}$ converges absolutely

(c) $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{e^{2n}}{n^n}\right)} = \lim_{n\to\infty}\left(\frac{e^2}{n}\right) = e^2 \lim_{n\to\infty}\left(\frac{1}{n}\right) = 0 < 1$

Converges by a root Test

(d) $\sum_{n=0}^{\infty} \frac{5^n}{n!}$ Let $a_n = \frac{5^n}{n!} \to a_{n+1} = \frac{5^n \cdot 5}{(n+1) \cdot n!}$

So $\lim_{n\to\infty}\left(\frac{a_{n+1}}{a_n}\right) = \lim_{n\to\infty}\left(\frac{5}{n+1}\right) = 0 < 1$

So $\sum \frac{5^n}{n!}$ converges by a ratio Test!

7. (**3pts**) Find a Maclaurin series for $h(x) = \sin(x)$ (<u>**show**</u> your work!).

$h^{(0)}(x) = \sin x \to h^{(0)}(0) = 0$

$h^{(1)}(x) = \cos x \to h^{(1)}(0) = 1$

$h^{(2)}(x) = -\sin x \to h^{(2)}(0) = 0$

$h^{(3)}(x) = -\cos x \to h^{(3)}(0) = -1$

$h^{(4)}(x) = \sin x \to h^{(4)}(0) = 0$

The series $\displaystyle\sum_{n=0}^{\infty} \frac{h^{(n)}(0)x^n}{n!} = \frac{x}{1!} + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} - \cdots$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

8. (**3pts**) <u>**Choose ONE**</u> of the following questions:(Either (a) OR (b) and clearly <u>**show**</u> your work!)

(a) Find the Maclaurin polynomial of order 4 for the function $f(x) = x^4 - 2x^3 + 4x^2 - 2x + 1$.

(b) Find the Maclaurin polynomial of order 4 for the function $f(x) = x^4 - 5x^3 - 2x^2 + 3x - 1$.

(a)
$$\left.\begin{array}{l} f(0) = 1 \\ f'(0) = -2 \\ f''(0) = 4 \\ f'''(0) = -2 \\ f^{(4)}(0) = 1 \end{array}\right\} \longrightarrow T_4(x) = 1 - 2x + 4x^2 - 2x^3 + x^4 = f(x)$$

(b)
$$\left.\begin{array}{l} f(0) = -1 \\ f'(0) = 3 \\ f^{(2)}(0) = -2 \\ f^{(3)}(0) = -5 \\ f^{(4)}(0) = 1 \end{array}\right\} \longrightarrow T_4(x) = -1 + 3x - 2x^2 - 5x^3 + x^4 = f(x)$$